Nowadays it is well-known that the Lorenz model is a paradigm one for low dimensional chaos in dynamical systems in synergetics and this model or its modifications are widely investigated in connection with modelling purposes in meteorology, hydrodynamics, laser physics,superconductivity,electronics,oil industry etc.,see,e.g.refs.1-15 and references therein. From the mathematical point of view, the Lorenz model is a system of nonlinear equations.Needless to say that in general it is vertually impossible to find a closed analytical solutions to the most of nonlinear equations.So one should take advantage of asymptotic approaches or have to recourse to the help of numerical simulations, which is not comprehensive for multi-parameter systems.In this paper we apply the asymptotic method for singularly perturbed nonlinear systems (ref.16 and references therein) to the Lorenz model.Earlier this method was applied by the author of this paper in refs.17-22 and references therein.
The system under study is of the form:

$$\frac{dx}{dt} = \sigma(y - x),$$

$$\frac{dy}{dt} = rx - y - xz, \qquad (10)$$

$$\frac{dz}{dt} = xy - bz,$$

where x,y and z are dynamical variables;σ ,r and b are the parameters of the sytem (1).In general initial conditions are : $x(t = 0) = x(0)$, $y(t = 0) = y(0)$, $z(t = 0) = z(0)$. Here we deliberately don't define the physical meaning of the dynamical variables and parameters, as in different fields of science the meaning is different. The system (1) will be investigated in the extreme cases, when σ^{-1} and b^{-1} are the small parameters of the problem.According to literature such values of parameters are quite possible (values $b >> 1$ are possible for non-laser systems ref.23).In order to apply the above-mentioned asymptotic method we rewrite the first equation of system (1) in the following form

$$\sigma^{-1}\frac{dx}{dt} = y - x, \qquad (2)$$

According to the theory in the zero-th order of σ^{-1} the solution of the system in the larger time domain is determined by the so-called reduced system :

$$y(t) = x(t),$$

$$\frac{dy}{dt} = rx - y - xz, \qquad (3)$$

$$\frac{dz}{dt} = xy - bz,$$

with the initial conditions $y(0)$, $z(0)$.In order to find the solution of the system (1) for small time domain (in the so-called boundary layer) we should make transition to the "new" time $\tau = \sigma t$,in other words

the time domain $t = 0$ is to be seen through microscope (as it is magnified σ times).After this operation for the solution of the system (1)in the boundary layer in the zero-th order of σ we obtain easily:

$$y(t) = y(0), z(t) = z(0),$$

$$x(t) = x(0)\exp(-\sigma t) + y(0)(1 - \exp(-\sigma t)), \qquad (4)$$

The solution of the system (1) in the whole time domain is to be constructed by linking (4) and solution to the system (3).The system (2) is the another nonlinear system to be studied by the asymptotic method in question.For these purposes we should make the following tranformations in the system (3):$y = b^{\frac{1}{2}} y_1$, $z = z_1$ and after that rewrite the third equation of the nonlinear system (3) in the following form:

$$b^{-1}\frac{dx_1}{dt} = y_1^2 - z_1, \qquad (5)$$

In other words system (3) will be studied on condition that $b^{-1}$is the small parameter.Acting as in the case of initial system (1) in the zero-th order of b^{-1} we easily obtain the solution of the system (3) in the whole time domain:

$$y(t) = b^{\frac{1}{2}} y_1,$$

$$z(t) = y_1^2 + (z_1(0) - y_1^2(0))\exp(-bt), \qquad (6)$$

If r is not equal to unity, then

$$y_1^2(t) = (r-1)\exp(2(r-1)t)(A + \exp(2(r-1)t))^{-1}, \qquad (7)$$

If $r = 1$ then

$$y_1^2(t) = (y_1^2(0) + 2t)^{-1}, \qquad (8)$$

In formulai (7) and (8)

$$A = ((r-1) - y_1^2(0))y_1^2(0), y_1(0) = b^{-\frac{1}{2}} y(0), \qquad (9)$$

Thus for the solution of the initial system (1) in the zero-th order of σ^{-1}, b^{-1} in the whole time domain we obtain:

$$x(t) = (x(0) - y(0))\exp(-\sigma t) + y_1(t)b^{\frac{1}{2}},$$

$$z(t) = y_1^2(t) + (z_1(0) - y_1^2(0))\exp(-bt),$$

$$y(t) = y_1(t)b^{\frac{1}{2}}, \qquad (10)$$

As the analysis of equations (10) show the characteristic time $t^{charact}$ of changing $y(t)$ from $y(0)$ to the stationary value of y, y^{stat} is of the order of $|(r-1)|$, if r is not equal to unity; if $r = 1$, then $t^{charact} \approx y_1^{-2}(0)$.Changing of $z(t)$ from $z(0)$ to z^{stat} occurs through the intermediary quasistationary state $z^{qstat} = y_1^2(0)$ with the required time to achieve this state $t^{qstat} = b^{-1}$; transition from z^{qstat} to z^{stat}

takes the amount of time equal to t^{charac}.The changing of $x(t)$ from $x(0)$ to x^{stat} occurs with the same scenario as for $z(t)$; the only difference is that the intermediary state for $x(t)$ is $y(0)$ with the transition time (from $x(0)$) $t^{tr} = \sigma^{-1}$.
In this paper we restricted ourselves to the case of zero-th order approximation.In the higher order approach we encountered with analytically hard treatable equations.The degree of adequacy of our formulai could be checked by the comparison with the behavior of the initial Lorenz model when independent variable t goes to infinity.Before comparing one should make clear that the asymptotic theory is not applicable when the nonlinear system develops full instability ref.4.This is the case for the Lorenz model, when for the given values of σ and b the value of r exceeds the so-called critical value (onset of chaotic behavior):

$$r_{cr} = (3 + b + \sigma)\sigma(\sigma - 1 - b)^{-1}, \qquad (11)$$

At $r > r_{cr}$ the non-zero fixed points (or steady states) of the Lorenz system

$$x^{stat} = y^{stat} = \pm(b(r-1))^{\frac{1}{2}},$$

$$z^{stat} = r - 1, \qquad (11)$$

become unstable,and there is a strange attractor over which a chaotic motion takes place. For $\sigma = 10$, $b = \frac{8}{3}$ the critical number is equal to $r_{cr} = 24.74$.Also it is known that the partial loss of instability in the Lorenz model occurs when $r > 1$:with this value of r the trivial steady state loses its stability ref.4. The analyses of our formulai show that indeed when $r > 1$ the system goes to the nontrivial steady state.In the contrary case the trivial steady state is obtained.Of course the presented here in this work results are of the simplest one for the Lorenz model which is capable to exhibit highly complicated behavior.But they are adequate at least for some extreme cases.
In conclusion in this work we investigate the Lorenz model in synergetics with the asymptotic method for singularly perturbed nonlinear systems for some limiting cases. The times of achieving quasistationary and stationary states are estimated.
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